CHARGE RADII AND MAGNETIC POLARIZABILITIES OF ρ and K^* MESONS IN QCD STRING THEORY

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Abstract

The effective action for light mesons in the external uniform static electromagnetic fields was obtained on the basis of QCD string theory. We imply that in the presence of light quarks the area law of the Wilson loop integral is valid. The approximation of the Nambu-Goto straight-line string is used to simplify the problem. The Coulomb-like short-range contribution which goes from one-gluon exchange is also neglected. We do not take into account spin-orbital and spin-spin interactions of quarks and observe the ρ and K^* mesons. The wave function of the meson ground state is the Airy function. Using the virial theorem we estimate the mean charge radii of mesons in terms of the string tension and the Airy function zero. On the basis of the perturbative theory, in the small external magnetic field we find the diamagnetic polarizabilities of ρ and K^* mesons: $\beta_\rho = -0.8 \times 10^{-4}$ fm^3, $\beta_{K^*} = -0.57 \times 10^{-4}$ fm^3

1 Introduction

One of the important problems of particle physics is the confinement of quarks. There is progress in understanding of properties of mesons as a system in which quarks and antiquarks are connected by the relativistic string with a Nambu-Goto self-interaction [1]. This binding interaction becomes strong at large distances and therefore it is impossible to describe the phenomenon using the perturbative approach. There are some difficulties in evaluating meson characteristics in the general case of a complicated string configuration. Naturally, as a first step, we make some approximations and model assumptions to simplify the calculations. So here we consider the straight-line string as a simple configuration and quarks attached to the ends of the string. Such configurations were studied in [2]. In the present paper we investigate mesons in external, constant, and uniform electromagnetic fields

and use the path integral approach. It should be noted that in potential-like models [3-5] meson characteristics are described reasonably. But in these approaches there are some assumptions: (i) the relativistic invariance is only the approximate, and (ii) the constituent quark masses are used (but not current quark masses).

The recent development of the QCD string approach [6-11] showed good results in describing heavy quarkonia, baryons and glueballs. The QCD string theory takes into account the main nonperturbative effects of strong interactions: chiral symmetry breaking (CSB) and the confinement of quarks. Chiral symmetry breaking gives a nonzero quark condensate ($\langle \bar{q}q \rangle$). As a result the light quarks (u, d - quarks) with current masses $m_u \simeq m_d \simeq 7$ MeV acquire the dynamical masses $\mu_u \simeq \mu_d \simeq 320$ MeV. This phenomenon is important for light pseudoscalar mesons as they possess the Nambu-Goldstone nature. To get low masses of pseudoscalar mesons one needs to take into account spin interactions of quarks. In [8,9] CSB was explained by the nonvanishing density of quark (quasi) zero modes in the framework of QCD. Then the familiar PCAC (partial conservation of axial vector current) theorems and the soft pion technique are reproduced. There are different stochastic vacuum configuration which are responsible for CSB: instantons (anti-instantons), pieces of (anti-)self-dual fields (for example, torons or randomly distributed lumps of field), and others. The necessary requirement is to have zero fermion modes. The condensation of zero modes leads to CSB. The confinement of quarks does not allow them to be observed; i.e. quarks cannot move outside of hadrons in large distances relative to each other. This was confirmed by Monte-Carlo simulations and experiments. Both nonperturbative effects of strong interactions can be explained by introducing stochastic gluon vacuum fields with definite fundamental correlators [6,7]. Then the linear potential between quarks appears and it provides the confinement of quarks. Besides, Regge trajectories are asymptotically linear with a universal slope [7]. So the method of vacuum correlators in nonperturbative QCD and the dynamics of zero modes give the explanation of the double nature of light pseudoscalar particles (pions, kaons and others) as Nambu-Goldstone particles and as the quark-antiquark system with the confining linear potential. It should be noticed that confinement prevents the delocalization of zero modes over the whole volume [8], i.e., stabilizes the phenomena of CSB.

In the present approach we make some assumptions. So we treats spin degrees of freedom as a perturbation and therefore it is questionable to apply this scheme to pions and kaons. Only ρ and K^* mesons are considered here

because the energy shift for them due to the hyperfine spin interaction is below 100 MeV [4]. Here short-range spin-orbital $\mathbf{L}\cdot\mathbf{S}$ and spin-spin $\mathbf{S_1}\cdot\mathbf{S_2}$ interactions are not taken into account. Besides we neglect the Coulomb-like short-range contribution due to the asymptotic freedom of QCD. This contribution is important only for heavy quarkonia [5]. We imply also that in the presence of light quarks the structure of the vacuum yields an area law of the Wilson loop integral. The restriction of the leading Regge trajectories is used as we consider here only ρ and K^* mesons.

It is important to calculate different intrinsic characteristics of hadrons on the basis of QCD string theory and to compare them with experimental values. It will be the test of this scheme. The charge radius (and electromagnetic form-factors) and electromagnetic polarizabilities of mesons are fundamental constants which characterize the complex structure of particles. These values for some mesons are known from the experimental data and therefore the estimation of them is reasonable. So we can check our notion about the vacuum structure by coinciding experimental data and theoretical predictions. In [12] we made the crude estimate of the charge radius and the electric polarizability of mesons and nucleons. Here we evaluate the mean-squared radius (using the virial theorem) and the magnetic polarizability of ρ and K^* mesons.

The electromagnetic polarizabilities of hadrons α, β enter the induced electric $\mathbf{D}=\alpha\mathbf{E}$ and magnetic $\mathbf{M}=\beta\mathbf{H}$ dipole moments, where $\mathbf{E}$, $\mathbf{H}$ are the strengths of electromagnetic fields. As a result there is a contribution to the polarization potential [13,14] as follows

$$U(\alpha,\beta)=-\frac{1}{2}\alpha\mathbf{E}^2-\frac{1}{2}\beta\mathbf{H}^2. \tag{1.1}$$

Electromagnetic polarizabilities are fundamental low-energy characteristics of strong hadron interactions and therefore they can be calculated in the framework of non-perturbative quantum chromodynamics $-$ QCD string theory.

This paper is organized as follows. In Sec. 2 after describing the general background we derive the effective action for mesons in external electromagnetic fields. The ground state and charge radii of particles are found on the basis of exact solutions and the virial theorem in Sec. 3. Section 4 contains the evaluation of the diamagnetic polarizabilities of ρ and K^* mesons using the perturbative expansion in a small magnetic field. In the conclusion we made a comparison of our results with other approaches.

Units are chosen such that $\hbar = c = 1$.

2 Effective Action for Light Mesons

To get the effective action for mesons in an external electromagnetic field we use the Green function of the quarks possessing spins in Minkowski space [see Eq. (A8) in the Appendix)]:

$$S(x,y) = i\int_0^\infty ds \int_{z(0)=y}^{z(s)=x} Dz \, \left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu\,(t)\right) P\exp\left\{ i\int_0^s \left[\frac{1}{4}\dot{z}_\mu^2(t) - m_1^2\right.\right.$$

$$\left.\left. + e_1\dot{z}_\mu(t)A_\mu^{el}(z) + \Sigma_{\mu\nu}\left(e_1F_{\mu\nu}^{el} + gF_{\mu\nu}\right)\right]dt\right\}\Phi(x,y), \tag{2.1}$$

where $z_\mu(t)$ is the path of the quark with the boundary conditions $z_\mu(0) = y_\mu$, $z_\mu(s) = x_\mu$ and

$$\Phi\,(x,y) = P\; exp\left\{ ig\int_y^x A_\mu dz_\mu\right\} \tag{2.2}$$

is the path-ordered product (the parallel transporter), A_μ is the gluonic field and g is the coupling constant. Neglecting quark-antiquark vacuum loops and omitting the annihilation graph, the Green function of mesons (the quark-antiquark system) takes the form [7]

$$G(x,\bar{x};y,\bar{y}) = \mathrm{tr}\left\langle \gamma_5 S(x,y)\Phi\,(y,\bar{y})\,\gamma_5 S(\bar{y},\bar{x})\Phi\,(\bar{x},x)\right\rangle, \tag{2.3}$$

were the brackets $\langle ...\rangle$ are the averaging over the external vacuum gluonic fields with the standard measure $\exp\left[iS\,(A)\right]$. Inserting (2.1) into (2.3) we find the expression

$$G(x,\bar{x};y,\bar{y}) = \mathrm{tr}\int_0^\infty ds\int_0^\infty d\bar{s}\int_{z(0)=y}^{z(s)=x} Dz\left(m_1 + \frac{i}{2}\gamma_\mu\dot{z}_\mu\,(t)\right)$$

$$\times\int_{\bar{z}(0)=\bar{y}}^{\bar{z}(\bar{s})=\bar{x}} D\bar{z}\left(m_2 - \frac{i}{2}\gamma_\mu\dot{\bar{z}}_\mu\,(\bar{t})\right)\; P_\Sigma\exp\left\{ i\int_0^s\left[\frac{1}{4}\dot{z}_\mu^2(t) - m_1^2 + e_1\dot{z}_\mu(t)A_\mu^{el}(z)\right.\right.$$

$$\left. + e_1\Sigma_{\mu\nu}F_{\mu\nu}^{el}\,(z)\right]dt + i\int_0^{\bar{s}}\left[\frac{1}{4}\dot{\bar{z}}_\mu^2(\bar{t}) - m_2^2 + e_2\dot{\bar{z}}_\mu(\bar{t})A_\mu^{el}(\bar{z}) + e_2\Sigma_{\mu\nu}F_{\mu\nu}^{el}\,(\bar{z})\right]d\bar{t}\right\}$$

$$\times\left\langle\exp\left\{ig\Sigma_{\mu\nu}\left[\int_0^s F_{\mu\nu}\,(z)\,dt - \int_0^{\bar{s}}F_{\mu\nu}\,(\bar{z})\,d\bar{t}\right]\right\}W(C)\right\rangle, \tag{2.4}$$

where P_Σ is the ordering operator of the spin matrices $\Sigma_{\mu\nu}$; e_1, e_2 are charges and m_1, m_2 are current masses of the quark and antiquark; $z_\mu(t)$, $\bar{z}_\mu(\bar{t})$ are the paths of the quark and antiquark with the boundary conditions $z_\mu(0) = y_\mu$, $z_\mu(s) = x_\mu$, $\bar{z}_\mu(0) = \bar{y}_\mu$, $\bar{z}_\mu(\bar{s}) = \bar{x}_\mu$ and $\dot{z}_\mu(t) = \partial z_\mu/\partial t$. Here we used the properties of γ matrices: $\{\gamma_5, \gamma_\mu\} = 0$, $[\Sigma_{\mu\nu}, \gamma_5] = 0$. As compared with [7,17] we added the interaction of charged quarks with the external electromagnetic fields. The gage - and Lorenz-invariant Wilson loop operator is given by

$$W(C) = \frac{\mathrm{tr}}{N_C} P \exp\left\{ ig \int_C A_\mu dz_\mu \right\}, \tag{2.5}$$

where N_C is the color number, and C is the closed contour of lines $x\bar{x}$ and $y\bar{y}$ connected by paths $z(t)$, $\bar{z}(\bar{t})$ of the quark and antiquark. The Wilson operator (2.5) contains both the perturbative and non-perturbative interactions between quarks via gluonic fields A_μ. In accordance with the approach [7], spin interactions can be treated as perturbations. It is justified for ρ and K^* mesons. To construct the expressions in spin interactions we write the relationship [7]

$$\left\langle \exp\left\{ ig\Sigma_{\mu\nu} \left[\int_0^s F_{\mu\nu}(z)\, dt - \int_0^{\bar{s}} F_{\mu\nu}(\bar{z})\, d\bar{t} \right] \right\} W(C) \right\rangle$$

$$= \exp\left\{ \Sigma_{\mu\nu} \left[\int_0^s dt \frac{\delta}{\delta\sigma_{\mu\nu}(t)} - \int_0^{\bar{s}} d\bar{t} \frac{\delta}{\delta\sigma_{\mu\nu}(\bar{t})} \right] \right\} \langle W(C) \rangle \tag{2.6}$$

where $\delta\sigma_{\mu\nu}(t)$ is the surface around the point $z_\mu(t)$. The zeroth order in spin-orbit and spin-spin interactions corresponds to neglecting the terms $g\Sigma_{\mu\nu}F_{\mu\nu}$ in Eq. (2.4). We suppose that mesons consist of quarks which move slowly with respect to the time fluctuations of the gluonic fields ($T_q \gg T_g$). It is the potential regime of the string. Voloshin [19] and Leutwyler [20] remarked that in another case ($T_q \ll T_g$) the dynamics is nonpotential and the QCD sum rules can be used. We consider the case when the distance between quarks $r \gg T_g$. Monte-Carlo calculations [21,22] gave $T_g \simeq 0.2 \div 0.3$ fm. So we imply that the characteristic quark relative distance is $r \simeq 1$ fm. This assumption will be confirmed below by the calculation of the quark-antiquark relative coordinate.

The average Wilson integral (2.5) at large distances in accordance with the area law can be represented in Minkowski space as

$$\langle W(C) \rangle = \exp(i\sigma_0 S), \tag{2.7}$$

where σ_0 is the string tension and S is the area of the minimal surface inside of the contour C. The surface S can be parametrized by the Nambu-Goto form [23,24]

$$S=\int_0^T d\tau\int_0^1 d\beta\sqrt{(\dot{w}_\mu w'_\mu)^2-\dot{w}_\mu^2 w'^2_\nu}, \tag{2.8}$$

where $\dot{w}_\mu=\partial w_\mu/\partial\tau$, $w'_\mu=\partial w_\mu/\partial\beta$. Using the approximation [7] that the coordinates of the string world surface $w_\mu(\tau,\beta)$ can be taken as straight lines for the minimal surface we write

$$w_\mu(\tau,\beta)=z_\mu(\tau)\beta+\bar{z}_\mu(\tau)(1-\beta), \tag{2.9}$$

where τ is implied to be the proper time parameter for both trajectories $\tau=(tT)/s=(\bar{t}T)/\bar{s}$. For uniform static external electromagnetic fields we have the representation of the vector potential through the strength tensor $F^{el}_{\mu\nu}$

$$A^{el}_\nu(z)=\frac{1}{2}F^{el}_{\mu\nu}z_\mu,\qquad A^{el}_\nu(\bar{z})=\frac{1}{2}F^{el}_{\mu\nu}\bar{z}_\mu. \tag{2.10}$$

The paths z_μ, $\bar{z}_\mu$ are expressed via the center of mass coordinate R_μ and the relative coordinate r_μ [7],

$$\bar{z}_\mu(\tau)=R_\mu-\frac{\bar{s}}{s+\bar{s}}r_\mu,\qquad z_\mu(\tau)=R_\mu+\frac{s}{s+\bar{s}}r_\mu \tag{2.11}$$

with the boundary conditions for $R_\mu(\tau)$, $r_\mu(\tau)$:

$$R_\mu(0)=\frac{\mu_1 y_\mu+\mu_2\bar{y}_\mu}{\mu_1+\mu_2},\qquad R_\mu(T)=\frac{\mu_1 x_\mu+\mu_2\bar{x}_\mu}{\mu_1+\mu_2},$$

$$r_\mu(0)=y_\mu-\bar{y}_\mu,\qquad r_\mu(T)=x_\mu-\bar{x}_\mu.$$

The integration with respect to z_μ, $\bar{z}_\mu$ in (2.4) is replaced by an integration over new variables R_μ, r_μ. As τ is a common time for the quark and antiquark (the time of the meson) the parametrization $z_\mu=(\tau,\mathbf{z})$, $\bar{z}_\mu=(\tau,\bar{\mathbf{z}})$ is possible [7]. This leads to the constraints: $R_0(\tau)=\tau$, $r_0(\tau)=0$. In accordance with the approach in [7] we introduce the dynamical masses μ_1, μ_2 by relationships

$$\mu_1=\frac{T}{2s},\qquad \mu_2=\frac{T}{2\bar{s}}. \tag{2.12}$$

Replacing the integration with respect to s, $\bar{s}$ in Eq. (2.4) by the integration over $d\mu_1$ and $d\mu_2$ with the help of (2.7) - (2.11) we find [12] the two-point function in zeroth order in the spin interactions

$$G(x,\bar{x};y,\bar{y}) = -T^2 \int_0^\infty \frac{d\mu_1}{2\mu_1^2} \int_0^\infty \frac{d\mu_2}{2\mu_2^2} \int DRDr\; exp\left\{iS_{eff}\right\} \tag{2.13}$$

with the effective action

$$\begin{aligned} S_{eff} = \int_0^T d\tau \Big[& -\frac{m_1^2}{2\mu_1} - \frac{m_2^2}{2\mu_2} + \frac{1}{2}\left(\mu_1+\mu_2\right)\dot{R}_\mu^2 + \frac{1}{2}\tilde{\mu}\dot{r}_\mu^2 \\ & + \frac{1}{2}F_{\nu\mu}^{el} e \left(\dot{R}_\mu R_\nu + \frac{1}{4}\dot{r}_\mu r_\nu\right) - \frac{q}{4}F_{\nu\mu}^{el}\left(\dot{R}_\mu r_\nu + \dot{r}_\mu R_\nu\right) \\ & - \int_0^1 d\beta \sigma_0 \sqrt{(\dot{w}_\mu w'_\mu)^2 - \dot{w}_\mu^2 w'^2_\nu}\Big], \end{aligned} \tag{2.14}$$

where

$$w_\mu = R_\mu + [\beta - \mu_1/(\mu_1+\mu_2)]r_\mu; \qquad \tilde{\mu} = \mu_1\mu_2/(\mu_1+\mu_2)$$

is the reduced mass of the quark-antiquark system, $e = e_1 + e_2$, $q = e_1 - e_2$. As a first step we are interested here in the spinless part and therefore the preexponential terms $\left(m_1 + \frac{i}{2}\gamma_\mu \dot{z}_\mu(t)\right)$, $\left(m_2 - \frac{i}{2}\gamma_\mu \dot{\bar{z}}_\mu(\bar{t})\right)$ and the constant matrix $\Sigma_{\mu\nu}F_{\mu\nu}^{el}$ was omitted in (2.13). The expression (2.14) defines the effective Lagrangian for light mesons in external uniform static electromagnetic fields in accordance with the formula $S_{eff} = \int_0^T \mathcal{L}_{eff} d\tau$. The expression (2.14) looks like a nonrelativistic one at $F_{\mu\nu}^{el} = 0$, but it is not. The author of [7] showed that the relativism is contained here due to the $\tilde{\mu}$ dependence and the spectrum is similar to that of the relativistic quark model.

The mass of the lowest states can be found on the basis of the relationship [25]

$$\begin{aligned} & \int DRDr \exp\left\{iS_{eff}\right\} \\ = & \left\langle R = \frac{\mu_1 x + \mu_2\bar{x}}{\mu_1+\mu_2}, r = x - \bar{x} \mid \exp\{-iT\mathcal{M}(\mu_1,\mu_2)\} \mid R = \frac{\mu_1 y + \mu_2 \bar{y}}{\mu_1+\mu_2}, r = y - \bar{y}\right\rangle, \end{aligned} \tag{2.15}$$

where the mass $\mathcal{M}(\mu_1,\mu_2)$ is the eigenfunction of the Hamiltonian. After that the Green function (2.13) is derived by integrating Eq. (2.14) over the dynamical masses μ_1, μ_2. In accordance with [7] we estimate the last

integration on $d\mu_1$, $d\mu_2$ using the steepest descent method which gives a good accuracy when the Minkowski time $T \to \infty$. To have the correct formulas, it is necessary to go into Euclidean space and return into Minkowski space on completing the functional integration. We use this procedure.

3 Ground State and Charge Radii

The last term in (2.14) can be approximated with the accuracy of ~ 5 % [7] by the relation

$$\int_0^1 d\beta \sqrt{(\dot{w}_\mu w'_\mu)^2 - \dot{w}^2_\mu w'^2_\nu}$$
$$= \int_0^1 d\beta \sqrt{\mathbf{r}^2 - \left(\beta - \frac{\mu_1}{\mu_1 + \mu_2}\right)^2 (\dot{\mathbf{r}} \times \mathbf{r})^2} \simeq \sqrt{\mathbf{r}^2}. \tag{3.1}$$

It is the potential regime at low orbital excitations of the string when the orbital quantum number l is small. As the equalities $R_0(\tau) = \tau$, $r_0(\tau) = 0$ are valid, only three-dimensional quantities are dynamical. Taking into account (3.1), from Eq. (2.14) using the standard procedure we find the canonical three-momenta corresponding to the center of mass coordinate R_μ and the relative coordinate r_μ

$$\Pi_k = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{R}_k} = (\mu_1 + \mu_2)\dot{R}_k + \frac{e}{2} F^{el}_{\nu k} R_\nu + \frac{q}{4} F^{el}_{\nu k} r_\nu,$$
$$\pi_k = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{r}_k} = \tilde{\mu}\dot{r}_k + \frac{e}{8} F^{el}_{\nu k} r_\nu + \frac{q}{4} F^{el}_{\nu k} R_\nu. \tag{3.2}$$

The Hamiltonian

$$\mathcal{H} = \pi_k \dot{r}_k + \Pi_k \dot{R}_k - \mathcal{L}_{eff}$$

found from Eq. (2.14) with the help of Eqs. (3.1), (3.2) takes the form

$$\mathcal{H} = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1 + \mu_2}{2} + \frac{\mu_1 + \mu_2}{2} \dot{R}_k^2 + \frac{\tilde{\mu}}{2} \dot{r}_k^2 - \frac{e}{2}(\mathbf{ER})$$
$$- \frac{q}{4}(\mathbf{Er}) + \sigma_0 \sqrt{\mathbf{r}^2} \tag{3.3}$$

so that the equation for the eigenvalues is given by

$$\mathcal{H}\Phi = \mathcal{M}(\mu_1, \mu_2)\Phi. \tag{3.4}$$

The terms containing the strength of the electric field in Eq. (3.3) describe the interaction of the dipole electric moment $\mathbf{d}$ with an external electric field. Using the definitions we have

$$\frac{e}{2}(\mathbf{ER}) + \frac{q}{4}(\mathbf{Er}) = \frac{1}{2}(e_1\mathbf{r_1} + e_2\mathbf{r_2})\mathbf{E} = \mathbf{dE} \tag{3.5}$$

and the interaction energy of the electric dipole moment with a uniform static electric field is $U = -\mathbf{dE}$. Bellow we investigate the case of a pure magnetic field when $\mathbf{E} = 0$. The case $\mathbf{E} \neq 0$, $\mathbf{H} = 0$ was considered in [12]. Using Eq. (3.2) the equation for the eigenfunction Φ of the auxiliary "Hamiltonian"

$$\tilde{\mathcal{H}} = \mathcal{H} - m_1^2/2\mu_1 - m_2^2/2\mu_2 - (\mu_1 + \mu_2)/2$$

is given by

$$\left[\frac{1}{2\tilde{\mu}}\left(\pi - \frac{e}{8}(\mathbf{r}\times\mathbf{H}) - \frac{q}{4}(\mathbf{R}\times\mathbf{H})\right)^2 + \frac{1}{2(\mu_1+\mu_2)}\left(\mathbf{\Pi} - \frac{e}{2}(\mathbf{R}\times\mathbf{H}) - \frac{q}{4}(\mathbf{r}\times\mathbf{H})\right)^2 + \sigma_0\sqrt{\mathbf{r}^2}\right]\Phi = \epsilon(\mu,\mathbf{H})\Phi, \tag{3.6}$$

where $\epsilon(\mu, \mathbf{H})$ is the eigenvalue. In accordance with the Noether theorem we come to the conclusion that the canonical momentum $\mathbf{\Pi}$ corresponding to the center of mass coordinate is a constant, i.e., $\mathbf{\Pi} = const$. Therefore it is possible to choose the condition $\mathbf{\Pi} = 0$. Putting $\mathbf{\Pi} = 0$ and $\mathbf{R} = 0$ into Eq. (3.6) we arrive to the equation

$$\left[\frac{1}{2\tilde{\mu}}\left(\pi - \frac{e}{8}(\mathbf{r}\times\mathbf{H})\right)^2 + \frac{q^2}{32(\mu_1+\mu_2)}(\mathbf{r}\times\mathbf{H})^2 + \sigma_0\sqrt{\mathbf{r}^2}\right]\Phi = \epsilon(\mu,\mathbf{H})\Phi. \tag{3.7}$$

The second term in (3.7) describes the effect of the recoil of the string. Such a term appears also in non-relativistic models [26,13,14]. If we put $\mathbf{R} = 0$ in Eq. (2.13), this term would not appear [12].

In quantum theory instead of the path integration in $\mathbf{r}$ we can use the replacement $\pi_k \to -i\partial/\partial r_k$. We can apply Eq. (3.7) to the leading trajectories with light quarks with masses $m_1 = m_2 \equiv m$, $\mu_1 = \mu_2 \equiv \mu$ ($\tilde{\mu} = \mu/2$) for ρ meson and when $m_1 \neq m_2$, $\mu_1 \neq \mu_2$ for K^* meson.

An external magnetic field splits the energy levels like the Zeeman effect for atoms. The difference with our case is we describe here the light quark-antiquark system in center mass system (c.m.s.) with the linear potential between quarks. Therefore the spectrum of the energy has other levels.

We can consider a small external magnetic field so that here perturbative theory can be applied. We receive a first approximation when the external field $\mathbf{H}$ is switched off ($\mathbf{H} = 0$) and the equation for the eigenvalue is given by

$$\left(-\frac{1}{2\tilde{\mu}}\frac{\partial^2}{\partial r_i^2} + \sigma_0\sqrt{\mathbf{r}^2}\right)\Phi = \epsilon(\mu)\Phi. \tag{3.8}$$

Equation (3.8) gives the discrete values of the energy $\epsilon(\mu)$ due to the shape of the potential energy. The numerical solution of equation (3.8) was obtained in [27]. It is useful to find the solution to Eq. (3.8) for the ground state in an analytical form. After introducing the variables $\rho_k = (2\tilde{\mu}\sigma_0)^{1/3} r_k$, $\epsilon(\tilde{\mu}) = (2\tilde{\mu})^{-1/3}\sigma_0^{2/3} a(n)$ [7], Eq. (3.8) becomes

$$\left(-\frac{\partial^2}{\partial \rho_i^2} + \rho\right)\Phi(\rho) = a(n)\Phi(\rho). \tag{3.9}$$

The solution to Eq. (3.9) may be chosen in the form $\Phi(\rho) = R(\rho)Y_{lm}(\theta, \phi)$, where $Y_{lm}(\theta, \phi)$ are spherical functions. After setting the variable $R(\rho) = \chi/\rho$ we come to the equation for the radial function

$$\chi^{''}(\rho) + \left(a(n) - \rho - \frac{l(l+1)}{\rho^2}\right)\chi(\rho) = 0, \tag{3.10}$$

where $\chi^{''}(\rho) = \partial^2\chi(\rho)/\partial\rho^2$, and l is an orbital quantum number. The solutions to Eq. (3.10) for the ground state $l = 0$ are the Airy functions $Ai(\rho - a(n))$, $Bi(\rho - a(n))$ [28]. The finite solution to Eq. (3.10) at $\rho \to \infty$ $(l = 0)$ is

$$\chi(\rho) = NAi(\rho - a(n)). \tag{3.11}$$

The constant N can be found from the normalization condition

$$\int_0^\infty \chi^2(\rho)d\rho = 1. \tag{3.12}$$

The requirement that this solution satisfies the condition

$$\chi(0) = NAi(-a(n)) = 0$$

gives the Airy function zeroes [28],

$$a(1) \equiv a_1 = 2.3381, \qquad a(2) \equiv a_2 = 4.0879$$

and so on. The principal quantum number $n = n_r + l + 1$, where n_r is the radial quantum number which defines the number of zeroes of the function $\chi(\rho)$ at $\rho > 0$. For the ground state we should take the solution (3.11) at $a(n) = a_1$ (here $n_r = 0$, $l = 0$):

$$\chi_0(\rho) = N_0 Ai(\rho - a_1). \tag{3.13}$$

Now let us estimate the mean-squared radius for the state which is described by the function Φ [the solution of equation (3.8)]. Multiplying Eq. (3.8) by the conjugated function Φ^* and integrating over the volume we find the relations

$$\langle T \rangle + \langle U \rangle = \epsilon(\tilde{\mu}),$$
$$\langle T \rangle = -\frac{1}{\mu} \int \Phi^* \partial_k^2 \Phi dV, \qquad \langle U \rangle = \sigma_0 \int \sqrt{\mathbf{r}^2} \Phi^* \Phi dV. \tag{3.14}$$

It is seen from Eqs. (3.14) that the mean potential energy $\langle U \rangle = \sigma_0 \left\langle \sqrt{\mathbf{r}^2} \right\rangle$ is connected to the mean diameter $\left\langle \sqrt{\mathbf{r}^2} \right\rangle$ (because $\mathbf{r}$ is the relative coordinate and quarks move around their center mass), which defines the size of mesons. In accordance with the virial theorem [29] we have the connection of the mean kinetic energy with the mean potential energy

$$2 \langle T \rangle = k \langle U \rangle, \tag{3.15}$$

where k is defined from the equality $U(\lambda r) = \lambda^k U(r)$. In our case of the linear potential $k = 1$ and from Eqs. (3.14), (3.15) we get

$$\langle U \rangle = \frac{2}{3} \epsilon(\tilde{\mu}) = \frac{2}{3} (2\tilde{\mu})^{-1/3} \sigma_0^{2/3} a(n). \tag{3.16}$$

The use of the steepest descent method for the estimation of the integration in μ (at $\mathbf{H} = 0$) leads to the conditions [7]:

$$\frac{\partial \mathcal{M}(\mu_1, \mu_2)}{\partial \mu_1} = 0, \qquad \frac{\partial \mathcal{M}(\mu_1, \mu_2)}{\partial \mu_2} = 0, \tag{3.17}$$

where the mass of the ground state $\mathcal{M}(\mu_1, \mu_2)$ is given by [see Eqs. (3.3), (3.4)]

$$\mathcal{M}(\mu_1, \mu_2) = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1 + \mu_2}{2} + (2\tilde{\mu})^{-1/3}\sigma_0^{2/3}a(n). \qquad (3.18)$$

Here we consider the more general case as compared with [7] when $\mu_1 \neq \mu_2$ $(m_1 \neq m_2)$. This case is realized for K^* mesons. It is assumed that the current mass of u,d quarks ($m_u = 5.6 \pm 1.1$ MeV, $m_d = 9.9 \pm 1.1$ MeV [30]), m_1 is much less than the dynamical mass μ_1 ($\mu_1 \simeq 330$ MeV), i.e., $m_1 \ll \mu_1$ and the mass of s quarks m_2 ($m_s = 199 \pm 33$ MeV [30]), is comparable with μ_1 but $m_2 < \mu_1$. Using these assumptions we neglect the term $m_1^2/2\mu_1$ in Eq. (3.18) and from Eqs. (3.17) have the equations

$$(2\tilde{\mu}\sigma_0)^{2/3}a(n) = 3\mu_1^2, \qquad 3m_2^2 + (2\tilde{\mu}\sigma_0)^{2/3}a(n) = 3\mu_2^2. \qquad (3.19)$$

From Eqs. (3.19) we arrive to the expression for the dynamical mass μ_2 (for s quarks):

$$\mu_2 = \sqrt{\mu_1^2 + m_2^2}. \qquad (3.20)$$

To find μ_1 the perturbation in the parameter m_2^2/μ_1^2 will be assumed. Using the relation $\mu_2 \simeq \mu_1(1 + m_2^2/(2\mu_1^2))$ which is obtained from Eq. (3.20) and the definition of the reduced mass $\tilde{\mu} = \mu_1\mu_2/(\mu_1 + \mu_2)$ from Eqs. (3.19) we arrive at the equation

$$\mu_1 \simeq \sqrt{\sigma_0}\left(\frac{a(n)}{3}\right)^{3/4}\left(1 + \frac{m_2^2}{8\mu_1^2}\right). \qquad (3.21)$$

In the zeroth order we come to the value $\mu_0 \equiv \mu_1^{(0)} = \sqrt{\sigma_0}(a(n)/3)^{3/4}$ [7]. The next order gives the relationship

$$\mu_1 \simeq \sqrt{\sigma_0}\left(\frac{a(n)}{3}\right)^{3/4} + \frac{m_2^2}{8\sqrt{\sigma_0}}\left(\frac{3}{a(n)}\right)^{3/4}. \qquad (3.22)$$

In a particular case $m_2 = 0$ we arrive at $\mu_1 = \mu_2 = \mu_0 = \sqrt{\sigma_0}(a(n)/3)^{3/4}$ [7]. The value of the string tension $\sigma_0 = 0.15$ GeV^2 was found from a comparison of the experimental slope of the linear Regge trajectories $\alpha' = 0.85$ GeV^{-2}, and the variable $\alpha' = 1/8\sigma_0$ [7]. It leads for the lowest state $n_r = 0, l = 0, a(1) = 2.3381$ to the value $\mu_0 = 321$ MeV [7]. This means that

for ρ mesons when $m_1 = m_u$, $m_2 = m_d$ we have the dynamical masses of u, d quarks $\mu_1 = \mu_2 = \mu_0$. For K^* mesons using Eq. (3.20) and $m_2 = m_s \simeq 200$ MeV [30] from Eq. (3.22) we get the reasonable values

$$\mu_1 \simeq 337 \text{ MeV}, \qquad \mu_2 \simeq 392 \text{ MeV}, \qquad \tilde{\mu} \simeq 181 \text{ MeV}. \tag{3.23}$$

Inserting the equation $\langle U \rangle = \left\langle \sigma_0 \sqrt{\mathbf{r}^2} \right\rangle$ into the left-hand side of Eq. (3.16) one gives the expression

$$\left\langle \sqrt{\mathbf{r}^2} \right\rangle = \frac{2}{3} (2\tilde{\mu}\sigma_0)^{-1/3} a(n). \tag{3.24}$$

From Eqs. (3.20), (3.22) using the first order in the parameter m_2^2/μ_1^2 we find

$$2\tilde{\mu} \simeq \mu_0 \left(1 + \frac{3m_2^2}{8\mu_0^2}\right) \qquad \left[\mu_0 = \sqrt{\sigma_0} \left(\frac{a(n)}{3}\right)^{3/4}\right]. \tag{3.25}$$

Equation (3.24) with the help of Eq. (3.25) gives an approximate relation for the mean relative coordinate:

$$\left\langle \sqrt{\mathbf{r}^2} \right\rangle = \frac{2}{\sqrt{\sigma_0}} \left(\frac{a(n)}{3}\right)^{3/4} \left[1 + \frac{3m_2^2}{8\sigma_0} \left(\frac{3}{a(n)}\right)^{3/2}\right]^{-1/3}. \tag{3.26}$$

For ρ mesons putting $m_2 = 0$ in Eq. (3.26) we arrive at

$$\left\langle \sqrt{\mathbf{r}^2} \right\rangle = \frac{2}{\sqrt{\sigma_0}} \left(\frac{a(n)}{3}\right)^{3/4}. \tag{3.27}$$

The same expression (3.27) was found in [12] using another method. With the help of the definition of the center of mass coordinate we can write an approximate relation for the mean charge radius of ρ-mesons: [1]

$$\sqrt{\left\langle r_\rho^2 \right\rangle} \simeq \frac{1}{2} \left\langle \sqrt{\mathbf{r}^2} \right\rangle. \tag{3.28}$$

At $\sigma_0 = 0.15$ GeV^2 [7,11] and $a(1) = 2.3381$ Eqs.(3.27), (3.28) give

$$\sqrt{\left\langle r_\rho^2 \right\rangle} \simeq 0.42 \text{ fm} \qquad \left(\left\langle \sqrt{\mathbf{r}^2} \right\rangle = 0.84 \text{ fm}\right). \tag{3.29}$$

[1] The relationship $\sqrt{\langle \mathbf{r}^2 \rangle} \simeq \left\langle \sqrt{\mathbf{r}^2} \right\rangle$ is confirmed by the numerical calculations.

The value (3.29) characterizes the radius of the sphere where the wave function of the ρ meson is concentrated (remember that $\mathbf{r}$ is the distance between quarks). We know only the experimental data for $\pi^{\pm}$ mesons which have the same quark structure as $\rho^{\pm}$ mesons:

$$\left\langle r^2_{\pi^{\pm}}\right\rangle_{exp} = (0.44 \pm 0.02)\ \mathrm{fm}^2 \qquad \left(\sqrt{\left\langle r^2_{\pi^{\pm}}\right\rangle_{exp}} \simeq 0.66\ \mathrm{fm}\right) \qquad [31].$$

For calculating the relative coordinate of K^* mesons we should use Eq. (3.24) or (3.26) with the conditions $\mu_1 = \mu_u$ and $\mu_2 = \mu_s$ Eqs. (3.23). As a result formula (3.24) gives the value of the mean relative coordinate of K^* mesons:

$$\left\langle \sqrt{\mathbf{r}^2}\right\rangle_{K^*} = 0.79\ \mathrm{fm}. \tag{3.30}$$

With the help of this relation we can estimate the mean charge radius of K^* mesons:

$$\sqrt{\langle r^2_{K^*}\rangle} \simeq \frac{\mu_2}{\mu_1+\mu_2}\left\langle \sqrt{\mathbf{r}^2}\right\rangle_{K^*} = 0.54\left\langle \sqrt{\mathbf{r}^2}\right\rangle_{K^*} = 0.43\ \mathrm{fm}. \tag{3.31}$$

The experimental data of the mean charge radius of $K^{\pm}$ mesons having an analogous quark structure as K^* mesons are

$$\sqrt{\langle r^2_{K^{\pm}}\rangle} = (0.53 \pm 0.05)\ \mathrm{fm} \qquad [32],$$

$$\left\langle r^2_{K^{\pm}}\right\rangle = (0.34 \pm 0.05)\ \mathrm{fm}^2 \qquad [31]$$

and for neutral K^0 mesons

$$\sqrt{\langle r^2_{K^0}\rangle} = (0.28 \pm 0.09)\mathrm{fm} \qquad [32].$$

So expression (3.26) gives a reasonable value for the charge radius of K^* mesons, although we need the experimental data of the charge radius of K^* mesons .

The first perturbative one-gluon exchange contribution to Hamiltonian (3.3) determines the spin-spin correction such as the Breit-Fermi hyperfine interaction [9]. The spin-spin interaction is important to explain the Nambu-Goldstone phenomenon which takes place for the $l = s = 0$ channel.

4 Perturbative Expansion and Magnetic Polarizabilities

To calculate the magnetic polarizability of mesons in accordance with Eq. (1.1) we should know the Hamiltonian depending on the magnetic field $\mathbf{H}$. From Eq. (3.7) we arrive at the expression of the auxiliary "Hamiltonian":

$$\tilde{\mathcal{H}} = -\frac{1}{2\tilde{\mu}}\frac{\partial^2}{\partial r_k^2} + \frac{e}{8\tilde{\mu}}\mathbf{HL} + \left(\frac{e^2}{128\tilde{\mu}} + \frac{q^2}{32(\mu_1+\mu_2)}\right)\left[\mathbf{r}^2\mathbf{H}^2 - (\mathbf{rH})^2\right] + \sigma_0\sqrt{\mathbf{r}^2}, \tag{4.1}$$

where $\mathbf{L} = -i(\mathbf{r}\times\partial)$ $(\partial_k = \partial/\partial r_k)$ is the angular momentum. Considering the external magnetic field $\mathbf{H} = (0,0,H)$ expression (4.1) is rewritten as

$$\tilde{\mathcal{H}} = -\frac{1}{2\tilde{\mu}}\frac{\partial^2}{\partial r_i^2} + \frac{eH}{8\tilde{\mu}}L_3 + \left(\frac{e^2}{4\tilde{\mu}} + \frac{q^2}{\mu_1+\mu_2}\right)\frac{H^2}{32}\left(r_1^2 + r_2^2\right) + \sigma_0\sqrt{\mathbf{r}^2}, \tag{4.2}$$

where $L_3 = i(r_2\partial_1 - r_1\partial_2)$ is the third projection of the angular momentum.

Now let us consider the magnetic polarizability of mesons on the basis of perturbative theory. We can rewrite Eq. (4.2) in the form

$$\tilde{\mathcal{H}} = \mathcal{H}_0 + \frac{eH}{8\tilde{\mu}}L_3 + \left(\frac{e^2}{4\tilde{\mu}} + \frac{q^2}{\mu_1+\mu_2}\right)\frac{H^2 r^2 \sin^2\theta}{32}, \tag{4.3}$$

where θ is the angle between $\mathbf{H}$ and the relative coordinate $\mathbf{r}$ and the free Hamiltonian is given by

$$\mathcal{H}_0 = -\frac{1}{2\tilde{\mu}}\frac{\partial^2}{\partial r_k^2} + \sigma_0\sqrt{\mathbf{r}^2}.$$

For the small magnetic fields $\mathbf{H}$ the second and third terms of Eq. (4.3) can be considered as a perturbation. Then using the standard perturbative method [33], we find a shift of the energy in the state $\mid n\rangle$ with the accuracy of second order in $\mathbf{H}$:

$$\Delta E_n = \langle n \mid \frac{eH}{8\tilde{\mu}}L_3 + \left(\frac{e^2}{4\tilde{\mu}} + \frac{q^2}{\mu_1+\mu_2}\right)\frac{H^2 r^2 \sin^2\theta}{32} \mid n\rangle$$

$$+\sum_{n'}\frac{\mid\langle n'\mid eHL_3/(8\tilde{\mu})\mid n\rangle\mid^2}{E_n-E_{n'}} \tag{4.4}$$

For the ground state $l=0$ (s state) the first and third terms of Eq. (4.4) do not give a contribution to the energy because $L_3\mid 0\rangle=0$. Taking the mean value and using the condition $(1/4\pi)\int\sin^2\theta d\Omega=2/3$, from Eq. (4.4) we come to

$$\Delta E_0=\left(\frac{e^2}{4\tilde{\mu}}+\frac{q^2}{\mu_1+\mu_2}\right)\frac{H^2}{48}\left\langle\mathbf{r}^2\right\rangle. \tag{4.5}$$

Comparing Eq. (4.5) with Eq. (1.1) we find the magnetic polarizability of light mesons

$$\beta=-\frac{1}{24}\left(\frac{e^2}{4\tilde{\mu}}+\frac{q^2}{\mu_1+\mu_2}\right)\left\langle\mathbf{r}^2\right\rangle. \tag{4.6}$$

It should be noticed that here $\langle\mathbf{r}^2\rangle$ means the mean-squared relative coordinate of the quark-antiquark system. Expression (4.6) is similar to the Langevin formula for the magnetic susceptibility of atoms. It is seen that we have here only the diamagnetic polarizability as $\beta<0$. To calculate the paramagnetic polarizability one needs to take into account the interaction of the meson spin with the magnetic field. Using the approximate relation $\langle\mathbf{r}^2\rangle\simeq\left\langle\sqrt{\mathbf{r}^2}\right\rangle^2$, parameters $e=e_1+e_2$, $q=e_1-e_2$, expression (3.27), and the dynamical masses of u,d quarks $\mu_1=\mu_2$, $\tilde{\mu}=\mu/2$ we find from Eq. (4.6) the relationship for the diamagnetic polarizability of ρ mesons:

$$\beta_\rho\simeq-\frac{(e_1^2+e_2^2)}{6\mu\sigma_0}\left(\frac{a(n)}{3}\right)^{3/2}, \tag{4.7}$$

Calculating Eq. (4.7) for charged ρ mesons at $\sigma_0=0.15$ GeV^2, $\mu=\mu_0=$ 321 GeV, $e_1=2e/3$, $e_2=e/3$, $e^2=1/137$, $n=1$ in Gaussian units one takes the value

$$\beta_\rho\simeq-0.8\times10^{-4}\ \text{fm}^3. \tag{4.8}$$

In rationalized units, the polarizability is 4π times greater.

For K^* mesons, Eq. (4.6) at the values (3.23), (3.31) $\left(\langle\mathbf{r}^2\rangle\simeq\left\langle\sqrt{\mathbf{r}^2}\right\rangle^2\right)$ leads to the magnitude

$$\beta_{K^*}=-0.57\times10^{-4}\ \text{fm}^3. \tag{4.9}$$

Unfortunately there are not experimental data of ρ and K^* meson polarizabilities yet.

5 Conclusion

The QCD string theory allows us to estimate the mean squared radii and magnetic polarizabilities of ρ, K^* mesons. These quantities were derived as functions of the string tension which is a fundamental variable in this approach. It is not difficult to calculate the magnetic polarizabilities of excited states of mesons using this approach. For that we should take the quantum numbers $n_r = 1$, $l = 0$ $(n = 2)$ and evaluate the mean relative coordinate in accordance with Eq. (3.26). Then Eq. (4.6) gives the necessary polarizabilities. To have more precise values of the meson electromagnetic characteristics one needs to take into account spin corrections. Especially it is important for light pseudoscalar mesons (π, K mesons). In principle it is possible to receive spin-orbit and spin-spin interactions using the general expressions (2.4), (2.6).

The Nambu-Jona-Lasinio (NJL) model [34] having a good basis in the framework of QCD [35] describes chiral symmetry breaking but not the confinement of quarks [7,36]. Besides this model has free parameters and the calculated polarizabilities of mesons [37] are parameter dependent.

The instanton vacuum theory (IVT) developed in [39-40] does not give the confinement of quarks phenomenon [7]. This theory is like the NJL model [36] and takes into account only chiral symmetry breaking. Therefore the calculation of the meson electromagnetic polarizabilities on the basis of the IVT gave the similar result [41] as in the NJL model.

All this shows that the theoretical evaluation of the charge radii and the magnetic polarizabilities of ρ, K^* mesons is possible on the basis of a good description of chiral symmetry breaking and the confinement of quarks in the framework of QCD string theory but with some approximations and model assumptions. Naturally that theory was derived using the nonperturbative QCD, i.e., first principals of QCD.

APPENDIX

In this appendix we derive the one-quark Green function using the Fock-Schwinger method. Starting with the approach [15] and introducing external electromagnetic and gluonic fields we write the Green function of quarks which possess spins in Minkowski space

$$S(x,y) = \left\langle x \mid \left(\widehat{D} + m_1\right)^{-1} \mid y \right\rangle = \left\langle x \mid \left(m_1 - \widehat{D}\right)\left(m_1^2 - \widehat{D}^2\right)^{-1} \mid y \right\rangle$$

$$= \left\langle \psi\left(x\right) \overline{\psi}\left(y\right) \right\rangle, \tag{A1}$$

where e_1 and m_1 are the charge and mass of the quark, $\widehat{D} = \gamma_\mu D_\mu$, $D_\mu = \partial_\mu - ie_1 A^{el}_\mu - igA_\mu$, $A_\mu = A^a_\mu \lambda^a$; γ_μ and λ^a are the Dirac and Gell-Mann matrices, respectively; A^{el}_μ and A^a_μ are the electromagnetic and gluonic vector potentials, respectively. The inverse operator $\left(m_1^2 - \widehat{D}^2\right)^{-1}$ can be represented in the proper time s [16]:

$$\left(m_1^2 - \widehat{D}^2\right)^{-1} = i \int_0^\infty ds \exp\left\{-is\left(m_1^2 - \widehat{D}^2\right)\right\}. \tag{A2}$$

Using the properties of Dirac matrices $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ we find the squared operator

$$\widehat{D}^2 = D_\mu^2 + \Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right), \tag{A3}$$

where

$$\Sigma_{\mu\nu} = -\frac{i}{4}[\gamma_\mu, \gamma_\nu], \qquad F^{el}_{\mu\nu} = \partial_\mu A^{el}_\nu - \partial_\nu A^{el}_\mu,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],$$

$\Sigma_{\mu\nu}$ are the spin matrices, and $F^{el}_{\mu\nu}$, $F_{\mu\nu}$ are the strength of electromagnetic and gluonic fields, respectively. Inserting relationship (A2) into Eq. (A1) with the help of Eq. (A3) we get

$$S(x,y) = i\int_0^\infty ds \langle x \mid \left(m_1 - \widehat{D}\right) \exp\Big\{-is\Big[m_1^2 - D_\mu^2$$
$$-\Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right)\Big]\Big\} \mid y\rangle. \tag{A4}$$

The exponent in Eq. (A4) plays the role of the evolution operator which defines the dynamics of the "Hamiltonian" $m_1^2 - D_\mu^2 - \Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right)$ with initial $\mid y\rangle$ and final $\langle x \mid$ states where s means the proper time. Therefore it is convenient to represent the matrix element in Eq. (A4) as a path integral [16]:

$$S(x,y) = i\int_0^\infty ds N \int_{z(0)=y}^{z(s)=x} DpDzP\left(m_1 - \widehat{D}\right) \exp\Big[i\int_0^s dt\Big[p_\mu \dot{z}_\mu - m_1^2$$
$$-\left(p_\mu - e_1 A^{el}_\mu - gA_\mu\right)^2 + \Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right)\Big]\Big], \tag{A5}$$

where $z_\mu(t)$ is the path of a quark with the boundary conditions $z(0)=y$, $z(s)=x$, $\widehat{D}=i\gamma_\mu\left(p_\mu-e_1A_\mu^{el}-gA_\mu\right)$ and P means ordering; N is a constant which is connected to the measure definition and it will be chosen later. The path integration over the momenta can be rewritten in the form (see [7,18]

$$N\int Dp\left(m_1-\widehat{D}\right)\exp\left\{i\int_0^s dt\left[p_\mu\dot{z}_\mu-\left(p_\mu-e_1A_\mu^{el}-gA_\mu\right)^2\right]\right\}$$
$$=N\int Dp\exp\left[i\int_0^s dt\left(p_\mu\dot{z}_\mu\right)\right]\left(m_1+\frac{1}{2}\gamma_\mu\frac{\delta}{\delta p_\mu}\right)$$
$$\times\exp\left\{-i\int_0^s dt\left(p_\mu-e_1A_\mu^{el}-gA_\mu\right)^2\right\}$$
$$=N\int Dp\left(m_1-\frac{i}{2}\gamma_\mu\dot{z}_\mu\right)\exp\left\{i\int_0^s dt\left[p_\mu\dot{z}_\mu-\left(p_\mu-e_1A_\mu^{el}-gA_\mu\right)^2\right]\right\}$$
$$=N\int Dp\left(m_1-\frac{i}{2}\gamma_\mu\dot{z}_\mu\right)\exp\left\{i\int_0^s dt\left[-p_\mu^2+\frac{1}{4}\dot{z}_\mu^2+\left(e_1A_\mu^{el}+gA_\mu\right)\dot{z}_\mu\right]\right\}. \tag{A6}$$

In Eq. (A6) we used the integration by parts (see [18]) and made a continuity of shifts $p_\mu\to p_\mu+e_1A_\mu^{el}+gA_\mu$ and then $p_\mu\to p_\mu+\dot{z}_\mu/2$. The constant N in Eq. (A6) is defined by the relation

$$N\int Dp\exp\left\{-i\int_0^s dt\left(p_\mu^2\right)\right\}=1. \tag{A7}$$

Taking into account Eqs. (A6), (A7) we find from Eq. (A5) the Green function of the quark

$$S(x,y)=i\int_0^\infty ds\int_{z(0)=y}^{z(s)=x}Dz\ \left(m_1-\frac{i}{2}\gamma_\mu\dot{z}_\mu(t)\right)P\exp\left\{i\int_0^s dt\Big[\frac{1}{4}\dot{z}_\mu^2(t)-m_1^2\right.$$
$$\left.+\left(e_1A_\mu^{el}+gA_\mu\right)\dot{z}_\mu(t)+\Sigma_{\mu\nu}\left(e_1F_{\mu\nu}^{el}+gF_{\mu\nu}\right)\Big]\right\}. \tag{A8}$$

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